Representation of Logical Form in Memory

Aaron W. Rader (rader.34@osu.edu)
Center for Cognitive Science and Department of Psychology
The Ohio State University, 208 Stadium East
1961 Tuttle Park Place, Columbus, OH 43210, USA

Vladimir M. Sloutsky (sloutsky.1@osu.edu)
Center for Cognitive Science, and School of Teaching & Learning
The Ohio State University, 208 Stadium East
1961 Tuttle Park Place, Columbus, OH 43210, USA

Abstract

Current theories of human deductive reasoning make different claims about the representation of logical statements in memory. Syntactically-based theories claim that abstract logical forms are represented veridically in memory, separate from content, whereas semantic theories propose that naïve reasoners represent combinations of possibilities that are based on the content of statements. We tested these predictions in two experiments in which participants had to recall and recognize statements of different logical forms. Results indicate that memory for logical form is not veridical, thus failing to support the syntactic view. In particular, results suggest that naïve participants tend, whenever possible, to represent only a single possibility for a statement of any logical form. These findings are consistent with semantic theories of human deductive reasoning and have significant implications for all theories of reasoning.

Introduction

The ability to reason and derive conclusions is ubiquitous in human life; however, naïve reasoning is error-prone (see Evans, Newstead, & Byrne, 1993; Evans & Over, 1996; Johnson-Laird, in press; Johnson-Laird & Byrne, 1991, for reviews). Because these errors exhibit systematic and predictable patterns, we believe that the analysis of these errors can shed light on the mechanism underlying human reasoning.

There are two major theoretical approaches to propositional reasoning, the syntax-based approach and the semantics-based approach. According to the syntactic approach, reasoners extract the syntactic form of the argument and apply certain formal rules of inference, or inferential schemata, to the extracted form (Braine & O'Brien, 1991; Rips, 1994). For example, reasoners easily conclude that *B* is the case, using the "modus ponens schema," when presented with the following premises:

$A \rightarrow B$ (If *A* then *B*)
A.

The syntactic approach thus hinges on assumptions that reasoners (a) veridically represent information in the premises and (b) apply inferential schemata to these representations. However, both assumptions are not uncontroversial. For example, according to the semantic approach, the untrained mind is not equipped with formal rules of inference. Furthermore, reasoning, to a large extent, is a function of representations of information in the premises. In turn, these representations are not veridical but are often incomplete or defective (Johnson-Laird & Byrne, 1991; Evans & Over, 1996; Sloutsky, Rader, & Morris, 1998).

In particular, when premises are compatible with multiple possibilities (e.g., *A* or *B*, or both), people tend to reduce the number of represented possibilities in accordance with the "principle of truth." The principle claims that people normally represent only true possibilities, and within these possibilities they represent only those propositions that are true (Johnson-Laird & Byrne, 1991; Yang & Johnson-Laird, in press). It has been also argued that people construe the "minimalist representation," discounting all possibilities except one (Sloutsky, et al. 1998; Sloutsky & Goldvarg, 1999).

If the assumptions of the syntax-based approach are true, then people should be able to extract logical form of a proposition and to construe a veridical representation of this form. If they do construe such veridical representations of the logical form, then, when the task is to remember these propositions, different logical forms should generate comparable error rates. Furthermore, because people tend to represent form, memory errors should be at least as likely to preserve the form, but not the content of propositions, as they should be to preserve the content but not the form.

However, if people do not represent propositions veridically, construing instead the "minimalist representation," then propositions that are compatible with one possibility (such as conjunctions) should generate fewer errors than propositions that are compatible with multiple possibilities (e.g., disjunctions, conditionals, or tautologies) or to no possibilities (such as contradictions). In addition, if they do not extract the logical form of the proposition, then, when committing errors, they should prefer foils that preserve the content, not the form of propositions. The two reported experiments were designed to test these predictions via examining recall and recognition of propositions varying in their logical form.

Experiment 1

Method

Participants Forty-nine undergraduate participants (35 females and 14 males, mean age = 23.0 years) from a large Midwestern university took part in the study. Some volunteered in return for extra credit, whereas others were paid a small cash amount for participation.

Materials A set of 18 pictures and 18 sentences were used in the experiment. Each picture, printed on plain white paper and laminated, measured approximately 4" by 5" and depicted a black-and-white line drawing of a face. The pictures were made to be visibly and distinctly different. Each sentence was printed in 14-point type on a slip of white paper, measured approximately 1.5" by 5", and was laminated. Each sentence was a description of a person; the descriptions always started with the phrase "This professor…" and then continued with either one or two clauses that used simple syntax (i.e., verb and direct object). The 6 one-clause sentences included 3 affirmations and 3 negations, and the 12 two-clause sentences included 3 each of the following logical forms: conjunction, disjunction, tautology and contradiction. In the learning phase (see below) each description was paired with one picture.

The other important materials included 18 lists of sentences, used in the recognition test, with one list corresponding to each description. Each list included that original description as well as 5 critical foils in which the contents of that description was phrased, as closely as possible, in the other five logical forms used in the study. For example, if the original description were a tautology, the list would include that description as well as its content presented as a contradiction, conjunction, disjunction, negation and affirmation. Note that in some cases, changing the form necessarily requires adding or deleting some content. For example, to represent a conjunction as an affirmation, we dropped the second proposition from each conjunction. The other 12 sentences on each list included six that used the same six forms, but content that appeared in none of the original 18 descriptions. Finally, six sentences with different content and forms were included on each list, as checks on guessing. The procedure also used a stopwatch to keep time.

Design and Procedure The experiment included two within-subject factors, Learning Trial and Logical Form of the sentence (affirmation, negation, conjunction, disjunction, tautology, and contradiction). Also, for the recognition test only, participants were divided into two groups, with one group tested on affirmations, conjunctions, and tautologies; and the other on negations, disjunctions, and contradictions. For each participant, the picture-sentence pairs were randomly determined, so that every participant received a unique pairing of pictures and descriptions. All participants were tested individually, seated at a small table across from the experimenter. The procedure was videotaped for subsequent analyses.

The experiment comprised three phases, a learning phase, a distracter phase, and a recognition test. Before the learning phase, the experimenter told each participant that she would be asked to try to remember a series of descriptions of people in response to pictures of those people. Participants also were told to pay attention to the exact wording of the statements and that some of them might sound odd. The learning phase consisted of five learning trials. On each trial, the participant first studied each of the picture-sentence pairs. A picture and its associated sentence were placed on the table in front of the participant, and the participant read the sentence aloud while studying the picture. Once the participant finished reading one description, the experimenter removed that picture-description pair and placed the next pair on the table. This procedure was repeated until all pairs had been presented, with the order of presentation randomized. The experimenter then placed one picture at a time on the table and asked the participant to recall its associated sentence. Again, order of presentation was random. After attempting to recall each picture, the participant repeated the learning trial sequence, until 5 trials were complete.

The next phase was a 5-minute distracter task, during which the participant completed math word problems. Following the distracter task, the participant was presented with the recognition test. Each participant was presented with 6 of the recognition lists, with two lists for each of the three forms in that participant's block (either affirmations, conjunctions, and tautologies; or negations, disjunctions, and contradictions). Every participant received a unique combination of lists, so that each description's list was given to a roughly equal number of participants, but no two participants received the same combination of lists. The experimenter informed the participant that she should decide, for each sentence on the list, whether it had been paired with a picture or not, and to place a check next to those sentences believed to have been presented earlier. Participants could take as much time as desired to finish this task, which was always completed in 3-5 minutes. Following this task the experimenter debriefed the participant.

Results and Discussion

Dependent variables for these analyses included participants' recalls in the learning phase and their choices in the recognition test. In all repeated measures analyses of variance to be reported in this and Experiment 2, only effects that were significant after applying the Geisser-Greenhouse correction to degrees of freedom in the F tests are reported. Additionally, all pairwise comparisons reported in both experiments were computed with the Bonferroni correction to the overall alpha rate of .05. Participants' gender is omitted because no significant differences emerged in preliminary analyses.

We first examined numbers of correct responses across trials and forms in the learning phase. A 5 (Trial) X 6 (Form) ANOVA, with repeated measures on both factors, yielded main effects of Trial, $F(4, 192) = 428.245$, $p < .001$; of Form, $F(5, 240) = 47.311$, $p < .001$; and a significant interaction, $F(20, 960) = 7.951$, $p < .001$. We also performed simple effects analyses of each Trial, following these with pairwise contrasts examining mean differences among Forms. These analyses revealed that, on Trial 1, atomic propositions and logical constants were recalled at higher rates than conjunctions and disjunctions, with the first four forms not differing from one another. On subsequent trials, however, the overall mean differences by form reported below were largely established. Consequently, and for the sake of brevity, we report differences among the overall means, shown in Figure 1.

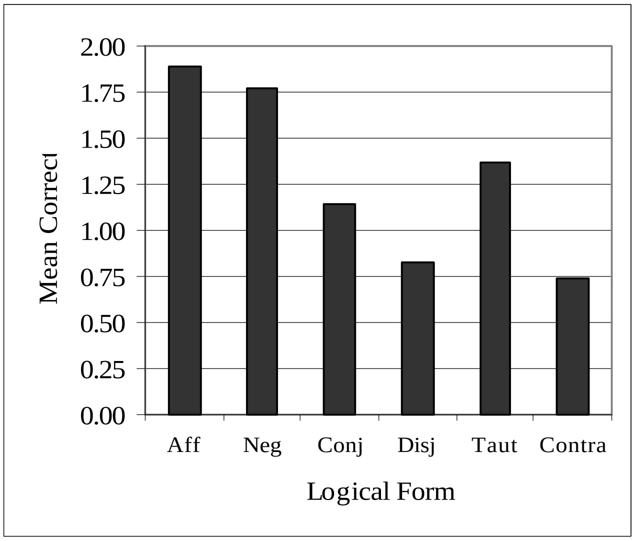

Figure 1: Mean correct recall by logical form, collapsed over trials.

Note that atomic statements (affirmations and negations) were recalled better than composite statements, which is not surprising given that they involve one proposition and no connective. Pairwise comparisons showed that affirmations (M = 1.89, SD = .50) were recalled significantly better than all other forms except negations (M = 1.78, SD = .45), all significant Fs(1,48) > 25, ps < .001. Negations, in turn, were also recalled at significantly higher rates than conjunctions (M = 1.14, SD = .51), disjunctions (M = .82, SD = .50), tautologies (M = 1.38, SD = .55) and contradictions (M = .74, SD = .64), all Fs(1, 48) > 50, ps < .001. Conjunctions were recalled at significantly higher rates than disjunctions, F(1, 48) = 4.84, p = .002; and contradictions, F(1, 48) = 8.00, p < .001. Recall rates for conjunctions and tautologies did not differ, F(1, 48) < 1, *NS*. Tautologies were recalled significantly more often than disjunctions and contradictions, Fs(1, 48) = 14.44 and 19.70, respectively, ps < .001. Finally, recall rates of disjunctions and contradictions did not differ significantly, F(1, 48) < 1, *NS*.

A second analysis focused upon recall errors involving substitutions of the logical connectives "and" and "or;" for example, recalling a conjunction as a disjunction by using "or." We refer to such errors as *conversions* because they convert a statement's logical form. Conversions occur when the content is correctly recalled, but the connective is not. This analysis used only recall attempts for the compound propositions, because such errors are not possible with atomic statements. A 5 (Trial) X 4 (Form) ANOVA, with repeated measures on both factors, revealed significant effects of Trial, F(4, 192) = 30.09, p < .001; of Form, F(3, 144) = 37.10, p < .001; and a Form X Trial interaction, F(12, 576) = 4.54, p < .001. We will focus on the Form effect, because inspection of the interaction showed it to be a function of contradictions' increasing likelihood of being converted over succeeding trials. Conversions for conjunctions, disjunctions, and tautologies did not increase nearly as much over trials, and the relative ordering of conversion rates of these forms was consistent; simple effects analyses of conversion rates by form on each trial revealed no significant cross-overs among these three forms. Means for each form are presented in Figure 2.

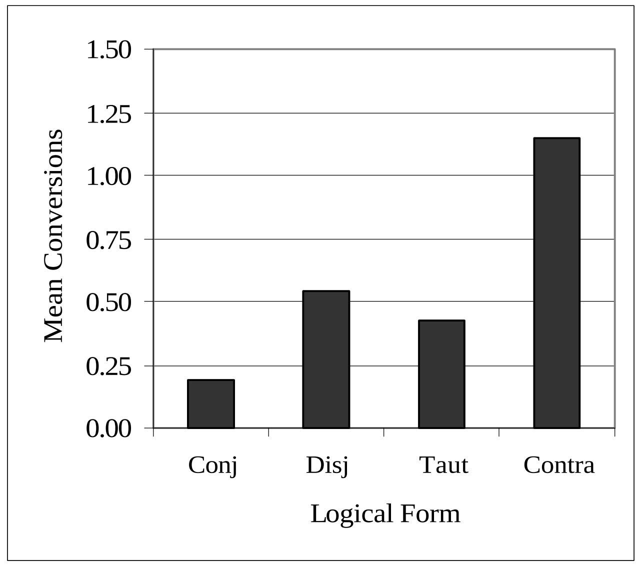

Figure 2: Mean conversions by logical form, collapsed over trials.

Pairwise comparisons following up the main effect of Form revealed that conjunctions (M = .19, SD = .25) were significantly less likely to be converted than disjunctions (M = .54, SD = .43), tautologies (M = .42, SD = .48), and contradictions (M = 1.15, SD = .61), all Fs(1, 48) > 8.2, ps < .006. Disjunctions and tautologies were both less likely to be converted than contradictions, Fs(1, 48) = 31.88 and 28.69, respectively, ps < .001. Finally, disjunctions and tautologies did not differ in their conversion rates.

We now turn to the recognition test results. In contrast to the recall data, these data indicate a substantial ability to remember the form and content of a statement. Separate binomial tests comparing the numbers of hits and false alarms for each form revealed that subjects' hit rates for affirmations (93%), negations (91%), conjunctions (71%), disjunctions (78%), and contradictions (64%) were significantly greater than false alarm rates, all zs >2.21, ps < .02. The hit rate for tautologies (55%) did not differ from the false-alarm rate, however, z = .80, p > .20. Inspections of false alarms revealed that 96% of all false alarms were those in which the content exactly matched the original statement but the connective was wrong (e.g., accepting a disjunction when the original statement was a conjunction). Participants' ability to select statements with precisely the original content was thus very good, and ability to select correct forms was also notable. These results disagree sharply with the learning phase data in two ways. First, no particular advantage exists for conjunctions relative to the other compound forms. Also, tautologies could not be discriminated from contradictions with the same content, whereas tautologies were recalled as well as conjunctions in the learning phase.

The learning phase data largely support our hypotheses and replicate earlier work with compound statements (Sloutsky, et al., 1998), but the recognition data raise questions. In recall, conjunctions were significantly more likely to be recalled than disjunctions and contradictions, and were less likely to be converted than all other compound forms. The lack of a difference between tautologies and conjunctions in recall was not predicted. Possibly, some participants may have been unsure of the connective used but were averse to producing contradictions (see Lakoff, 1971). The finding that atomic statements were easier to recall than all compound statements is not surprising given that atomic statements do not involve a connective and are shorter.

The recognition data not only indicated overall better memory (this finding is trivial on its own) but also indicated that the relative accuracies in memory for tautologies and contradictions switched, compared to recall. We suspected that the overall improvement and perplexing changes with tautologies and contradictions partially stemmed from the recognition test itself (i.e., presenting separate lists for each statement, with only one correct answer per list). In Experiment 2, we made the recognition test somewhat more complex by testing participants for all original statements. We also assessed recognition performance after varying numbers of learning trials because five trials allow many opportunities for rote memorization. With these modifications we could better test our original hypothesis with respect to recognition memory.

Experiment 2

Method

Participants Forty-five undergraduate participants (25 female and 20 male, mean age = 19.3 years) from a large Midwestern university volunteered for the experiment in partial fulfillment of a course requirement. Fifteen participants were assigned to each Learning Trial condition.

Materials All materials were the same as Experiment 1 except the recognition test. The new test included the original 18 descriptions and the five foil statements associated with each description from the lists of Experiment 1. The test also included the six foil statements associated with each description that involved the same logical forms but different content. Overall, the test consisted of 216 items. The different-content, different-form distracters from Experiment 1 were not included because results of that study and a pilot study of this experiment, using only one learning trial, showed that these items were never chosen. The test was presented as an eight-page typewritten packet, and sentences were randomly ordered.

Design and Procedure The experiment included one within-subject factor, Logical Form, and one between-subject factor, Number of Learning Trials (1, 3, or 5). Instructions to participants were the same as Experiment 1. The learning phase also followed the same procedure, with each participant receiving either one, three, or five trials. The distracter and recognition phases were the same as Experiment 1.

Results

Gender is again omitted because no such differences emerged. We initially conducted a 2 (Number of Trials: 3 or 5) X 6 (Form) mixed ANOVA on number of correct recalls during the learning phase on the last trial for each group. The 1-trial group was omitted because, perhaps owing to a smaller sample size, it contributed virtually no variance (participants in this group largely recalled nothing, and in the recognition test they chose many of the different-content foils, indicating that the task was perhaps too difficult for them). The recall data parallel those in Experiment 1, so we do not report them here.

For the recognition results, we analyzed accuracy for each logical form, operationalized for each participant as (Hits - False alarms)/3. Such a computation puts accuracy on a scale ranging from –1 to +1, which was appropriate because overall frequencies of acceptances for each form varied somewhat. Accuracy varies within the range [-1, +1] because every participant chose only some combination of correct choices and same-content, wrong-connective foils; no other distracters were chosen. Accuracy data are presented in Figure 3.

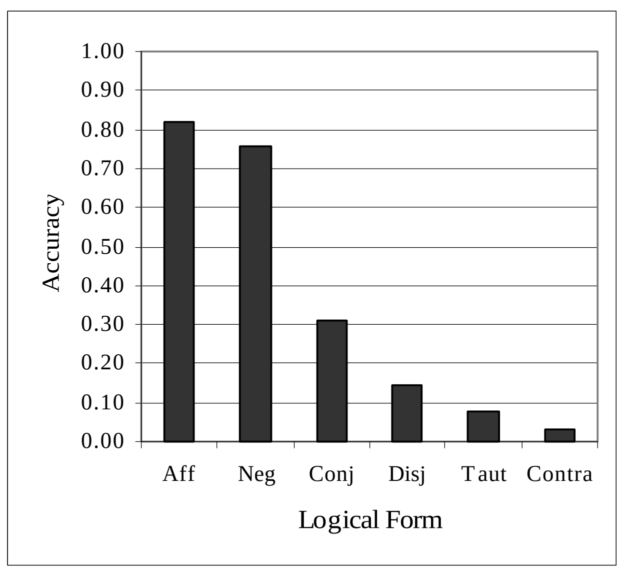

Figure 3: Mean recognition accuracy by form.

A 2 (Number of Trials) X 6 (Form) ANOVA on accuracy yielded only a main effect of Form, $F(5, 140) = 16.65$, $p < .001$. Number of trials and the interaction were not significant, $F(1, 28) = 1.21$ and $F(5, 140) = 1.02$, respectively, ps > .25. Pairwise comparisons of these means revealed higher accuracies for affirmations (M = .82, SD = .29) than for all compound forms, all Fs(1, 28) >37, ps < .001. Negations (M = .75, SD = .30) also had higher accuracies than all compound forms, all Fs(1, 28) > 30, ps < .001. Affirmations and negations did not differ significantly. No mean differences among conjunctions (M = .31, SD = .47), disjunctions (M = .14, SD = .61), tautologies (M = .08, SD = .49), or contradictions (M = .03, SD = .65) were significant. For each form, t-tests comparing mean accuracy to 0 showed that accuracy was significantly greater than zero for affirmations and negations, ts(29) > 13, ps < .001. Mean accuracy for conjunctions was also significantly greater than zero, $t(29) = 3.64$, $p = .001$. For the other three forms, accuracy did not differ significantly from zero, all ts < 1.28, ps > .2.

General Discussion

Taken together, both experiments offer some support to the hypothesis that reasoners tend to construct a single, conjunctive representation of possibilities for compound propositions. In Experiment 1, conjunctions were more accurately recalled than all other compound forms except tautologies, and participants converted conjunctions less often than all other compound forms. The advantage for conjunction in recall also supports the idea that conjunctions, being more informative than other compound forms, should be easiest to learn (see Sloutsky, et al., 1998). Tautologies were recalled as accurately as conjunctions, but were nevertheless more likely to be converted. As discussed earlier, the high recall rate for tautologies (and the lower rate for contradictions) may partially have resulted from an aversion to producing contradictory statements when the correct connective was in doubt. Recognition of, and aversion to, contradiction is believed to be a basic part of logical competence (Macnamara, 1986). In Experiment 2, although mean recognition accuracies for all compound forms were lower than those for atomic forms, and did not differ from one another, only accuracy for conjunctions was significantly greater than zero. These data suggest that participants may have been less likely to false alarm on same-content, wrong-connective foils with conjunctions than with other compound forms. Such a pattern would be expected if conjunctions are easier to commit to memory and involve less uncertainty in a recognition task.

Our data do not support the claim that naïve reasoners possess a stock of syntactic inference schemata in long-term memory (Rips, 1994). Such a theory would have to predict, for these tasks, that memory errors altering a statement's logical form should be randomly distributed over forms because all forms (except possibly tautologies and contradictions) should be available. The pattern of memory errors, at least in recall, also questions the claim that people automatically parse sentences for their logical forms (Braine, 1990), again because conjunctions seem to be privileged with respect to other compound forms. A critic might claim that disjunctions, tautologies, and contradictions are less acceptable than conjunctions and may be subject to "normalization" in comprehension; that is, they may be altered into the more accepted format (Fillenbaum, 1977). This view cannot predict, however, differences in recall among these forms, which we found. Also, even some conjunctions have no "common topic" that makes them sensible (Lakoff, 1971, p. 116). Because disjunctions and conjunctions in these experiments were intentionally constructed to contain two unrelated propositions, pragmatic explanations cannot easily account for the recall results.

These results do not necessarily impugn the claim that parsing into logical form occurs in an autonomous, on-line fashion (Lea, 1995); possibly, syntactic abstraction does occur but logical syntax is quickly discarded in favor of a semantic representation. However, our experiments make the claim for mental logic more elusive. Investigations of recognition at varying intervals after presentation, from immediate test to substantial delays, will help settle the issue, as will investigations into whether logical inferences putatively represented as procedural rules respond to experimental manipulations in ways consistent with rule-based, autonomous processes (Smith, Langston, & Nisbett, 1992). We are currently conducting both types of investigations.

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