Children's acquisition of fraction knowledge from concrete versus generic instantiations

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Abstract

The goal of this experiment was to investigate elementary school children's ability to acquire basic fraction knowledge. The degree of concreteness of instantiations of proportions was varied between subjects. First-grade children learned to label proportions of objects with fraction. Proportions were presented either as concrete, colorful flowers or as generic black and white circles. Following instruction, participants were given a test of learning and an immediate or delayed test of transfer involving proportions of novel objects. Those who learned with the generic materials scored higher on learning and transfer than those who learned with the concrete materials. Differences between learning conditions were attenuated for the delayed transfer test. These findings suggest that concrete, perceptually rich instantiations of fractions may hinder children's acquisition of basic fraction knowledge in comparison to simple generic instantiations of fractions.

Keywords: Psychology; Education; Learning, Transfer; Relations, Mathematics Education.

Introduction

Mathematical concepts are often difficult for children to acquire. One response to this challenge is to introduce concepts to students through concrete instantiations which include perceptually rich, familiar material. The use of concrete material is widespread in education (see McNeil & Uttal, 2009 for discussion). Concrete instantiations of mathematics may involve familiar contexts and can be visually appealing and engaging. For example, simple arithmetic concepts are often instantiated through sets of familiar objects, such as two apples plus three apples equals five apples. Such material may spark interest in the learning task and maintain attention on the learning material.

However, a primary goal of learning mathematics is the ability to apply mathematics to new situations. Therefore, successful acquisition of mathematical knowledge implies that the learner has not only acquired knowledge of the mathematical relations in the context of learning, but also has the ability to transfer the mathematical knowledge to novel isomorphic situations. There is evidence that concrete instantiations can hinder transfer of learning. Adults who learned a novel mathematical concept from a generic, perceptually sparse instantiation were better able to transfer this knowledge to a novel isomorph than those who learned the same concept from a concrete instantiation (Kaminski, Sloutsky, & Heckler, 2008; Sloutsky, Kaminski, & Heckler, 2005; see also Goldstone & Sakamoto, 2003; Goldstone & son, 2005 for related findings).

In comparison to more abstract, generic instantiations, concrete instantiations of a given concept communicate more extraneous information. For example, a photograph of a person communicates more nonessential information than a simple stick figure drawing (see Kaminski & Sloutsky, 2011 for a discussion). Similarly, instantiating addition as the sum of apples communicates more information than instantiating it as the sum of tally marks. This additional information (e.g. the appearance, taste, etc. of apples) is extraneous to the mathematics and may present an obstacle for learning for the following reason. Mathematical concepts are defined by relational structure. Relations are less salient than objects (e.g. Gentner, 1988). Instantiating mathematical concepts through concrete material, in comparison to a more generic format, may increase the salience of superficial aspects of the learning material and consequently divert the learner's attention from the to-be-learned relational structure (see Goldstone, Medin, & Gentner, 1991 for a similar argument regarding similarity judgments), thus hindering learning.

Therefore, it appears that generic instantiations of mathematical concepts have an advantage over concrete instantiations with respect to transfer. However it could be argued that this advantage is limited to older learners. After all, in educational practice, older students are expected to learn and reason with abstract instantiations including symbols, equations, and other standard notation. It may be that younger learners (e.g. elementary school students) may need concrete instantiations to begin to acquire abstract knowledge.

In addition to the notion that concrete material may be more engaging for the learner, support for the use of concrete instantiations to teach abstract concepts to young children is often tied to theories of learning and development. Some developmental theories (Montessori, 1917; Piaget, 1970) posit that young children's thinking is inherently concrete and that they are not capable of reasoning about abstract concepts using symbols. According to these theories, children proceed through developmental stages in which their reasoning becomes more abstract and less dependent on concrete material. Other theories (e.g., Bruner, 1966) tie the ability to reason about abstract material not to developmental stages but to levels of knowledge. From this perspective, all novices, regardless of age, would need concrete instantiations to begin to acquire knowledge of an abstract concept. Both accounts suggest that learning should begin by instantiating the mathematical concept through concrete, familiar material.

However, if the difficulty transferring knowledge from concrete instantiations is due to extraneous information diverting attention from the relevant relational structure, then concrete instantiations may be at least as detrimental for children's learning as they are for adults. Children have difficulty controlling their attentional focus and filtering irrelevant, potentially distracting information (Kemler, 1982; Shepp & Swartz, 1976; Smith & Kemler, 1978, see also Hanania & Smith, 2010). For example, Shepp and Swartz (1976) instructed 6- and 9-year-olds to sort items according to shape, with color being an irrelevant dimension. It was found that 6-year-olds (but not 9-year-olds) were slower when color varied independently of shape than when color co-varied with shape or did not vary at all. Therefore, the task-irrelevant dimension affected performance of younger, but not older participants.

There is also evidence that concrete, perceptually rich material can hinder preschool children's ability to perform simple relational tasks in comparison to performance with generic material. One line of evidence comes from studies of children's early symbol use. Successful symbol use requires the detection and transfer of common relations. For example, to effectively use a map as a symbol for a real location, one must recognize the common relations between entities on the map and their real-world analogs. Two- and three-year-old children are more successful transferring location information from a picture to the real world than from a 3-dimensional scale model to the real world (DeLoache, 1995a, 1995b). These findings suggest that preschool children have difficulty using concrete, perceptually rich objects as symbols than using less concrete objects as symbols.

In addition, preschool children are better able to detect relations of monotonic increase and monotonic decrease in size between displays that involve simple perceptually sparse objects than concrete, perceptually rich objects (Gentner, Ratterman, Markman, & Kotovsky, 1995; Kaminski, & Sloutsky, 2010). It has also been demonstrated that kindergarten children are better able to recognize common proportions across displays of different objects when first given instruction with generic, perceptually sparse objects than when given instruction with concrete, perceptually rich objects (Kaminski & Sloutsky, 2009). Taken together there is evidence that concrete, perceptually rich material may hinder the recognition of relational structure for kindergarteners and preschool children.

Less is known about how concreteness affects young school-aged children's acquisition of relational knowledge that is part of standard mathematics content. While concrete instantiations of mathematics may communicate distracting extraneous information, it is possible that school-aged children have developed sufficient inhibitory control to focus on the relevant relations and not be distracted by extraneous aspects of concrete material. If this is the case, then concrete instantiations will not hinder learning of mathematical concepts and may even facilitate learning by making the material more interesting for children. However, while executive function is maturing throughout childhood, the complexity of the relations we expect children to learn is increasing.

Higher-order mathematical concepts involve more complex relations than simpler mathematical concepts. For example, the concept of addition is relationally more complex than the concept of set cardinality (i.e. the use of a natural number to represent the number of elements in a set). Preschool children learn the concept of set cardinality, while school-age children learn the concept of addition which entails determining the cardinality of the union of two sets. Similarly, the concept of multiplication is relationally more complex than the concept of addition because multiplication is defined as repeated addition. It may be that when children reach a level of development at which they are capable of attending to some relations in the context of extraneous information, they may not be able to attend to more complex relations in the presence of the same extraneous information. As a result, the acquisition of more complex relations from concrete instantiations may be more susceptible to diverted attention than acquisition of simpler relations. Therefore, we propose that concreteness in the presence of more complex relations, such as arithmetic relations, can hinder knowledge acquisition in comparison to more generic instantiations of the same concepts.

Overview

The purpose of the present research was to test the hypothesis that concreteness of the learning material will hinder young school-aged children's acquisition of mathematical knowledge. The present study examined initial learning and subsequent transfer of basic fraction knowledge when instruction involved either a concrete, perceptually rich instantiation versus a generic, perceptually sparse instantiation. First-grade students were taught to label proportions of discrete objects with fractions. Transfer was measured as students' ability to label proportion of novel objects with fractions. For half the participants, transfer was tested immediately after instruction. For the other half of participants, transfer was tested after a two-week delay.

Experiment

Method

Participants Participants were 64 first-grade students recruited from middle-class, suburban schools in the Columbus, Ohio area (34 girls and 30 boys, M = 7.3 years, SD = .40 years).

Materials and Design The experiment had a 2 (Learning condition: Concrete vs. Generic) by 2 (Transfer Test: Immediate vs. Delayed) between-subjects design. Participants were randomly assigned to one of the two learning conditions and one of the two transfer test times. The timing of the transfer test was a between-subject factor to control for any potential testing effects on delayed transfer.

The task was to label proportions of discrete objects with fractions. The experiment had two phases. The first phase consisted of training and a test of learning. Training consisted of four examples of how to label a proportion of objects with a fraction, followed by six questions with corrective feedback. In the Generic condition, all training examples were proportions of black circles out of black and white circles. In the Concrete condition, all training examples were proportions as purple flowers out of purple and orange flowers. Figure 1 presents one of the examples used in training for both the Generic and Concrete conditions.

Following training, participants were given an eight-question test of learning which presented novel proportions in the same format as the training (i.e. circles for the Generic condition and flowers for the Concrete condition). Questions were multiple-choice. Four questions presented a proportion and participants were asked to select a fraction that described the proportion. The remaining four questions presented a fraction and participants were asked to select a collection of objects for which the proportion matched the fraction. Four response choices were given: (1) the correct response, (2) correct numerator, but incorrect denominator, (3) correct denominator, but incorrect numerator, and (4) incorrect numerator and incorrect denominator. The order of the answer choices was counterbalanced across question trials.

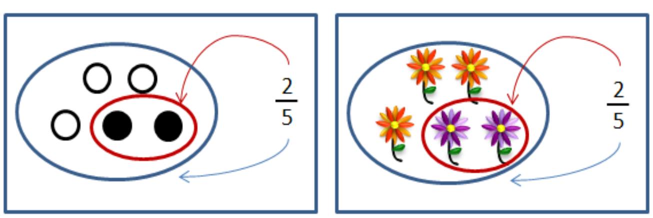

Figure 1: Example of labeling a proportion with a fraction from the training phase (Generic condition on left, Concrete condition on right).

The second phase of the experiment was a transfer task in which participants were given 24 multiple-choice questions involving novel objects. For half of the participants the transfer test was given immediately after phase 1 (i.e. training and testing of learning) and for the other half of participants the transfer test was given two weeks after phase 1. Twelve questions presented a collection of objects and participants selected a fraction that described a specified proportion of the objects (see Figure 2); the other twelve questions presented a fraction and participants selected a corresponding collection of objects that showed a proportion matching the fraction (see Figure 3). For each question, there were four possible response choices, as described for the training questions. The questions involved proportions with denominators (i.e. total number of items in a display) ranging from 2 to 10. Many different items were used for response choices and included: red and blue cars, blue and green shoes, red and green fish, green and red bugs, bears with and without flags, cupcakes with and without sprinkles, slices of pizza (present or missing), light windows and dark windows of a house, partially full bus seats, partially full pencil box, partially full paint bucket and partially remaining chocolate bar.

Procedure All training and test questions were presented on the computer. During training, the experimenter gave a definition of proportion and explained that fractions can describe proportions. For example, in the Generic condition when showing the example of 2/5 (see Generic condition of Figure 1), the experimenter stated while gesturing to the circles, "The proportion of black circles in this group is 2/5 because there are five circles all together, 1, 2, 3, 4, 5, and two of them are black, 1, 2". Explanations in the Concrete condition were completely isomorphic to those of the Generic condition. Participants proceeded through the test questions at their own pace. The experimenter recorded their responses through the computer.

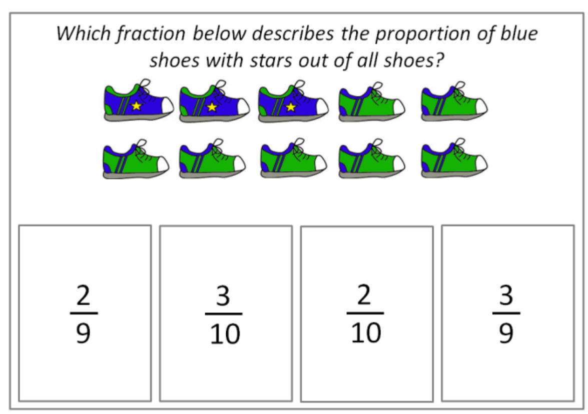

Figure 2: Example of a transfer test question for which participants needed to choose a fraction that matched the proportion shown.

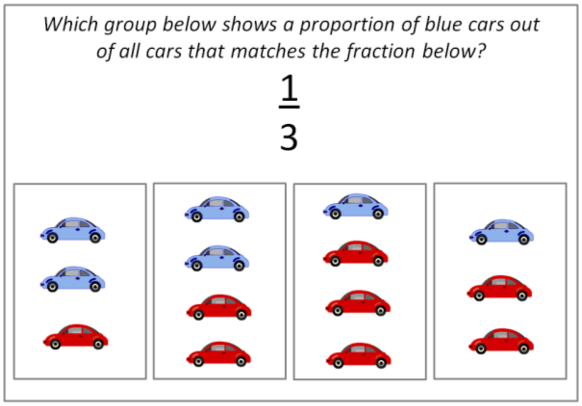

Figure 3: Example of a transfer test question for which participants needed to choose a proportion that matched the fraction shown.

Results

Two participants, one from the Concrete Delayed condition and one from the Generic Immediate condition, were removed from the analysis because their learning scores were more than 2.5 standard deviations below the mean score in their conditions.

In both the Concrete and Generic conditions, children successfully learned. Learning scores in both conditions were well above a chance score of 25% (see Figure 4), one-sample t-tests, *t*s > 18.0, *p*s < 0.001. However, participants in the Generic condition scored significantly higher than those in the Concrete condition (M = 92.1%, SD = 10.1% for Generic and M = 81.2%, SD = 17.7% for Concrete), independent samples t-test, $t(60) > 2.93$, $p < .006$, Cohen's $d = .752$

Transfer test scores were also above chance in both learning conditions (Concrete and Generic) and transfer test time conditions (Immediate and Delayed) (see Figure 4), one-sample t-tests *t*s > 13.0, *p*s < 0.001. However, there was a significant difference in transfer scores between the learning conditions on the immediate transfer test, independent samples *t*s(29) > 2.39, $p < .024$, Cohen's $d = .895$. Participants in the Generic condition scored higher than those in the Concrete condition. The difference in transfer scores attenuated considerably on the delayed transfer test, independent samples $t(29) = .704$, $p = .487$, Cohen's $d = .200$. An analysis of variance was performed with transfer test score as the dependent variable, learning condition and transfer test time as fixed factors and learning score as a covariate. The results reveal a significant effect of learning score, $F(1, 57) = 24.2$, $p < .001$, $\eta_p^2 = .298$, and no significant effect of learning condition, $F(1, 57) = .366$, $p = .548$, and no effect of transfer time, $F(1, 57) = .815$, $p = .371$. There was a moderate interaction between learning condition and transfer time $F(1, 57) = 2.90$, $p = .094$ (see Figure 4). For both immediate and delayed testing, transfer scores were

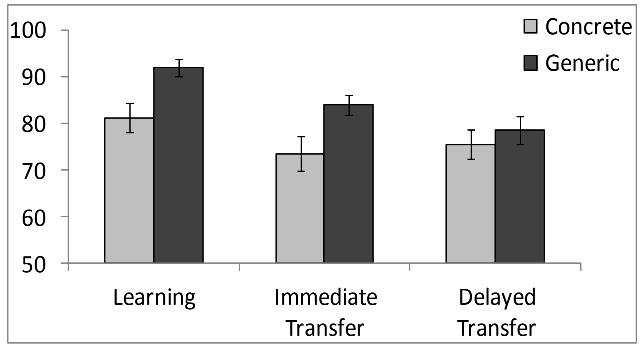

Figure 4: Mean Test Scores (% Correct).
Note: Error bars represent standard error of mean.
Chance score is 25%.

positively correlated with learning scores, Pearson Correlations, $r(29) = .600$, $p < .001$ and $r(29) = .554$, $p < .002$ respectively. These findings suggest that transfer is a function of learning such that higher levels of learning result in higher levels of transfer.

Taken together the results of this experiment suggest that learning basic fraction knowledge from a concrete instantiation, in comparison to a more generic instantiation, can hinder initial learning which may in turn hinder subsequent transfer to novel material. With time, the negative effect of concreteness on transfer appears to be attenuated.

General Discussion

This research considered first-grade children's ability to acquire basic knowledge of the concept of fraction. Participants were instructed on how to label proportions of objects with fractions. Instruction presented proportions either through generic black and white shapes or through colorful, familiar objects. Participants were tested on their ability to label proportions of novel objects with fractions. Participants who received instruction with either type of material successfully learned and applied their knowledge to novel objects. However, those who were instructed with the generic instantiation scored 10% higher on tests of learning and immediate transfer to novel objects than those who were instructed with the concrete instantiation. The difference in transfer scores due to instruction with the concrete versus generic learning material diminished when the transfer test was delayed for two weeks. Yet both immediate and delayed transfer test scores were strongly correlated with learning scores.

The results of this study support the hypothesis that concreteness of the learning material can hinder children's acquisition of mathematical knowledge. In particular, it appears that instruction involving concrete instantiations of proportions hinders initial learning and consequently hinders subsequent transfer in comparison to instruction involving generic instantiations of proportion. Instruction of basic fraction notation using generic material may help students gain a solid knowledge foundation which may in turn benefit them when learning more advanced mathematical concepts involving fractions. Although concrete instantiations are often colorful and visually appealing, bland, generic instantiations are learnable by children and can offer an advantage for learning and subsequent transfer of mathematical knowledge.

These findings suggest that although aspects of executive function, including the ability to control attentional focus and inhibit irrelevant information, mature considerably in the preschool years, extraneous information included in concrete educational material may be difficult for elementary school children to ignore. Learning of mathematical concepts may be hindered because less attentional resources have been allocated to the relevant to-be-learned relations.

With respect to actual pedagogical practice, mathematics instruction is generally not limited to using only one instantiation of a concept and frequently involves multiple instantiations, including formal symbolization as well as familiar contextualization. Concrete and abstract instantiations of mathematical may both have advantages. However, it is not clear a priori when and how to include concrete instantiations and generic instantiations in instruction. For example, there appears to be a trade-off between grounded, concrete instantiations and abstract, symbolic instantiations when solving algebra problems where grounded, concrete formats facilitate solving simple problems and abstract, symbolic formats facilitate solving more complex problems (Koedinger, Alibali, & Nathan, 2008; Koedinger & Nathan, 2004). The results of the present study provide evidence of an advantage for generic material for acquiring knowledge of basic fraction notation. The challenge for researchers and educators is to develop a theoretical basis for the timing and use of both concrete and generic instantiations in instruction of mathematical concepts in general.

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